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ABSTRACT

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Two scattering problems involving a moving medium are studied in this report. The first deals with the reflection of a plane electromagnetic wave from a semi-infinite moving medium. The second is an investigation of the scattering field from a rotating dielectric cylinder. The formulation is based upon the Maxwell-Minkowski equations which are accurate to the order of v/c, where v denotes the velocity of a moving medium and c the velocity of light.

Author

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TWO SCATTERING PROBLEMS INVOLVING MOVING MEDIA

I. INTRODUCTION

Numerous boundary-value problems involving moving media have not yet been investigated. The lack of such a study is probably due to the fact that an exact formulation of this class of problems requires a relativistic treatment which is usually quite complicated. Recently, a digest of Minkowski's theory of electrodynamics of moving, isotropic, linear, media has been presented[1]. It was shown that if the velocity of the moving medium, v, is small compared to the velocity of light, c, the intricate relations contained in Minkowski's theory can be greatly simplified. Thus, by neglecting terms of the order of $(v/c)^2$, one may obtain the following set of equations for the electromagnetic field vectors $\vec{E}$ and $\vec{H}$:

$$\nabla \times \vec{E} = - \frac{\partial}{\partial t} (\mu \vec{H} - \vec{\Lambda} \times \vec{E}) \tag{1}$$

and

$$\nabla \times \vec{H} = \sigma(\vec{E} + \mu \vec{v} \times \vec{H}) + \frac{\partial}{\partial t} (\epsilon \vec{E} + \vec{\Lambda} \times \vec{H}) , \tag{2}$$

where

σ, μ, ϵ = conductivity, permeability, and permittivity of an isotropic linear medium at rest;

$\vec{\Lambda} = (\epsilon\mu - \epsilon_0\mu_0)\, \vec{v}$,

and

$\vec{v}$ = velocity of the moving medium.

A number of boundary-value problems based upon Eqs. (1) and (2) have already been solved. Thus, wave propagation in wave-guides filled with a moving medium have been studies by Collier[2]. The radiation of an antenna in a moving medium has been studied by Compton and Tai[3]. In this report, the solutions for two scattering problems will be presented. The first deals with reflection and transmission of a plane wave incident upon a semi-infinte moving medium. The second deals with the scattering of a plane wave from a rotating dielectric cylinder.

II. REFLECTION OF A PLANE WAVE FROM A SEMI-INFINITE MOVING MEDIUM.

Let us consider a plane wave which is propagating in free space and is incident obliquely upon a semi-infinite moving medium, as shown in Fig. 1.

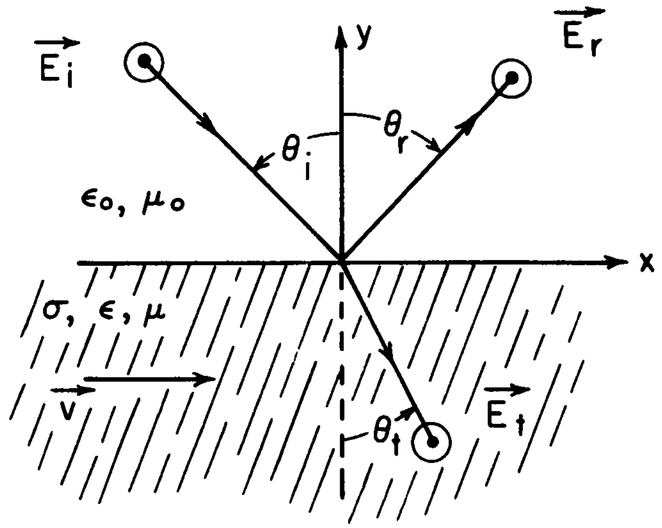

Fig. 1. Reflection of a plane wave from a semi-infinite moving medium.

The electric field of the incident wave is assumed to be perpendicular to the plane of incidence, hence it has only a z-component, which is given by

$$(3) \qquad \mathbf{E}_i = E_i \, e^{ik_o(x \sin \theta_i - y \cos \theta_i)} ,$$

where

$$k_o = \omega(\mu_o \epsilon_o)^{\frac{1}{2}} = 2\pi/\lambda .$$

It is understood that the time factor for the harmonically oscillating field is $e^{-i\omega t}$. The expression of the electric field for the reflected wave then may be written in the form

$$\mathbb{E}_r = E_r \, e^{ik_o(x \sin \theta_r + y \cos \theta_r)} . \tag{4}$$

To determine the proper expression for the transmitted wave, a brief review is necessary concerning the plane wave solutions pertaining to Eqs. (1) and (2) which were discussed in great detail in Ref. (2). For harmonically oscillating fields, Eqs. (1) and (2) may be reduced to

$$(\nabla + i\omega \vec{\Lambda}) \times \vec{E} = i\omega\mu \vec{H} \tag{5}$$

and

$$(\nabla + i\omega\vec{\Lambda} - \sigma\mu \, \vec{v}) \times \vec{H} = (\sigma - i\omega\epsilon) \, \vec{E} . \tag{6}$$

By taking the divergence of Eqs. (5) and (6), respectively, and simplifying the results, one may obtain the following two auxiliary equations:

$$(\nabla + i\omega \vec{\Lambda}) \cdot \vec{H} = 0 \tag{7}$$

and

$$(\nabla + i\omega\vec{\Lambda} - \sigma\mu \, \vec{v}) \cdot \vec{E} = 0 . \tag{8}$$

By eliminating $\vec{H}$ between Eq. (5) and Eq. (6), making use of Eq. (8), and neglecting terms of the order of $(v/c)^2$, the following equation for $\vec{E}$ is obtained:

$$[\nabla^2 + (2i\omega\vec{\Lambda} - \sigma\mu \, \vec{v}) \cdot \nabla + k^2(1 + i\delta)] \, \vec{E} = 0, \tag{9}$$

where

$$k = \omega(\mu\epsilon)^{\frac{1}{2}} = \omega/c$$

and

$$\delta = \sigma/\omega\epsilon .$$

If we let

(10) $$\vec{E} = E_t \, e^{i\gamma(x \sin \theta_t - y \cos \theta_t)} \hat{z}$$

represent the transmitted or the refracted wave, then by substituting Eq. (10) into Eq. (9) the following characteristic equation for γ results:

(11) $$-\gamma^2 + i\gamma \sin \theta_t (2i\omega\Lambda - \sigma\mu v) + k^2(1 + i\delta) = 0 \, .$$

Introducing the constant m, defined by

(12) $$m = 1 - \frac{\mu_o \epsilon_o}{\mu \epsilon} \quad ,$$

so that

(13) $$\Lambda = (\mu\epsilon - \mu_o\epsilon_o) v$$
$$= \mu\epsilon m v \, ,$$

Eq. (11) can then be written in the form

(14) $$\gamma^2 + k \sin \theta_t \left(\frac{v}{c}\right) (2m + i\delta)\gamma - (1 + i\delta) = 0 \, ,$$

where

$$c = 1/\sqrt{\mu\epsilon} \, .$$

The solution for γ, neglecting terms of the order of $(v/c)^2$, is given by

(15) $$\gamma = k \left\{ \frac{1}{\sqrt{2}} [(1 + \sqrt{1 + \delta^2})^{\frac{1}{2}} + i(-1 + \sqrt{1 + \delta^2})^{\frac{1}{2}}] - \left(m + i\frac{\delta}{2}\right) \frac{v}{c} \sin \theta_t \right\} \, .$$

For the case that $\delta \ll 1$, Eq. (15) reduces to

$$(16) \qquad \gamma = k\left[\left(1 - m\frac{v}{c}\sin\theta_t\right) + i\frac{\delta}{2}\left(1 - \frac{v}{c}\sin\theta_t\right)\right].$$

In this report we shall discuss the simplest case where the moving medium is lossless, that is, $\delta = 0$. The general expressions for the $\vec{E}$ and $\vec{H}$ fields of the transmitted wave are then given by

$$(17) \qquad \vec{E}_t = E_t\, e^{i\gamma(x \sin\theta_t - y\cos\theta_t)}\, \hat{z}$$

and

$$(18) \qquad \vec{H}_t = \frac{1}{i\omega\mu}(\nabla + i\omega\vec{\Lambda}) \times \vec{E}_t$$

$$= \frac{E_t}{i\omega\mu}\left[-i\gamma\cos\theta_t\,\hat{x} - (i\omega\Lambda + i\gamma\sin\theta_t)\,\hat{y}\right] \cdot e^{i\gamma(x\sin\theta_t - y\cos\theta_t)},$$

where

$$(19) \qquad \gamma = k\left(1 - m\frac{v}{c}\sin\theta_t\right).$$

By matching the boundary conditions at the interface, $y = 0$, for the $\vec{E}$-field and the x-component of the $\vec{H}$-field, the following equations are obtained:

$$(20) \qquad k_o \sin\theta_i = k_o\sin\theta_r = \gamma\sin\theta_t\,,$$

$$(21) \qquad E_i + E_r = E_t\,,$$

and

$$(22) \qquad \frac{k_o}{\mu_o}\left[\cos\theta_i E_i - \cos\theta_r E_r\right] = \frac{\gamma}{\mu_o}\cos\theta_t E_t\,.$$

Equation (20) implies that

$$\sin \theta_i = \sin \theta_r \tag{23}$$

and

$$\frac{\sin \theta_i}{\sin \theta_t} = \frac{\gamma}{k_o} = \frac{k}{k_o}\left(1 - m\,\frac{v}{c}\sin \theta_t\right). \tag{24}$$

Snell's Law relating the angle of incidence and the angle of refraction is, therefore, slightly modified as a result of the motion of the semi-infinte medium. By solving Eq. (24) for $\sin \theta_t$ in terms of $\sin \theta_i$, and keeping only the term accurate to v/c, we find

$$\sin \theta_t = \frac{k_o}{k}\sin \theta_i \left[1 + m\left(\frac{k_o}{k}\right)\left(\frac{v}{c}\right)\sin \theta_i\right]. \tag{25}$$

As far as the reflection and the transmission amplitudes are concerned, by solving Eqs. (21) and (22) for E_r and E_t, one finds that the Fresnel formula is still unchanged, i.e.,

$$2E_o = \left[1 + \frac{\mu_o}{\mu}\,\frac{\tan \theta_i}{\tan \theta_t}\right] E_t \tag{26}$$

and

$$2E_r = \left[1 - \frac{\mu_o}{\mu}\,\frac{\tan \theta_i}{\tan \theta_t}\right] E_t\ . \tag{27}$$

For the case $\mu = \mu_o$ (moving dielectric), Eqs. (26) and (27) can be combined to give

$$E_o : E_r : E_t = \sin(\theta_t + \theta_i) : \sin(\theta_t - \theta_i) : [\sin(\theta_t + \theta_i) + \sin(\theta_t - \theta_i)]. \tag{28}$$

The analysis described here can easily be extended to a moving slab. Again, the only change, as compared to a stationary slab, will be the angle of refraction. In the case of a lossy moving medium, the formal analysis is the same except that the expressions for γ given by Eq. (15) or Eq. (16) should be used in Eqs. (20) and (22).

III. SCATTERING OF A PLANE WAVE BY A ROTATING DIELECTRIC CYLINDER

Strictly speaking, the Maxwell-Minkowski equations, as described by Eqs. (1) and (2), are valid only when $\vec{v}$ is constant, which corresponds to a pure translation. However, according to Sommerfeld[4], we may consider these equations to be valid even for non-uniform $\vec{v}$, provided that the acceleration due to the change of $\vec{v}$ is not great. It is safe to assume here that until one can justify Sommerfeld's assertion by means of a more rigorous study based upon the general theory of relativity, we shall treat the formulation of the problem discussed in this section as an approximate one subject to more rigorous investigation.

The problem to be discussed is illustrated in Fig. 2, where a plane wave is incident upon a rotating dielectric cylinder. We are interested to find the scattered field.

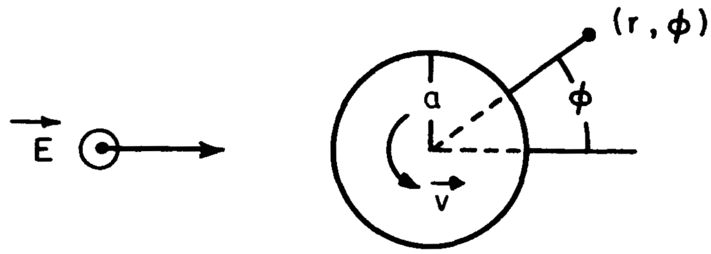

Fig. 2. A plane wave incident upon a rotating dielectric cylinder.

It is assumed that angular velocity of the rotating cylinder is constant, being equal to ω_c. The $\vec{E}$-field of the incident wave is assumed to be polarized in the direction of the axis of the cylinder. The incident wave is, then, given by

$$(29) \qquad E_i = E_o \, e^{ik_o r \cos\phi}$$

$$= E_o \sum_{-\infty}^{\infty} (i)^n J_n(k_o r) \, e^{in\phi} \, .$$

The scattering field can be written in the form

$$E_s = E_o \sum_{-\infty}^{\infty} \alpha_n (i)^n H_n^{(1)}(k_o r)\, e^{in\phi}. \tag{30}$$

To find the transmitted field inside the dielectric cylinder, we shall first solve Eqs. (5) and (6) under the condition that the field vectors are independent of z and

$$\vec{v} = \omega_c r \hat{\phi} . \tag{31}$$

The parameter $\vec{\Lambda}$ is, then, a function of r and it may be written in the form

$$\begin{aligned} \vec{\Lambda} &= \mu_o(\epsilon - \epsilon_o)\, \vec{v} \\ &= \mu_o \epsilon \left(1 - \frac{\epsilon_o}{\epsilon}\right) \omega_c r \hat{\phi} \\ &= K r \hat{\phi} , \end{aligned} \tag{32}$$

where

$$K = \mu_o \epsilon \left(1 - \frac{\epsilon_o}{\epsilon}\right) \omega_c = \frac{m \omega_c}{c^2} , \quad \text{with} \tag{33}$$

$$m = 1 - \frac{\epsilon_o}{\epsilon} , \quad c^2 = 1/\mu_o \epsilon .$$

By substituting Eq. (32) into Eqs. (5) and (6) and eliminating $\vec{H}$, we obtain a differential equation for E_z which is the only component of the electric field inside the rotating cylinder. The resultant equation for E_z is given by

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial E_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 E_z}{\partial \phi^2} + 2i\omega K \frac{\partial E_z}{\partial \phi} + k^2 E_z = 0, \tag{34}$$

where

$$k^2 = (\omega/c)^2 .$$

In deriving Eq. (34), we have neglected a high-order term corresponding to $\omega^2 K^2 r^2 E_z$, which is proportional to $(v/c)^2$. It should be emphasized that the Maxwell-Minkowski equations as given by Eqs. (5) and (6) are only accurate to the order v/c, hence all terms of the order of $(v/c)^2$ should be omitted from the final result.

To solve Eq. (34) we let

$$E_z = e^{in\phi} F(r). \tag{35}$$

The function F(r) then satisfies the Bessel equation

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dF}{dr}\right) + \left(k^2 - 2n\omega K - \frac{n^2}{r^2}\right) F = 0 . \tag{36}$$

If we introduce the constant γ_n, such that

$$\begin{aligned} \gamma_n{}^2 &= k^2 - 2n\omega K \\ &= k^2 - 2n\omega m\left(\frac{\omega_c}{c^2}\right) \\ &= k^2\left(1 - \frac{2nm\omega_c}{\omega}\right), \end{aligned} \tag{37}$$

then the proper set of radial functions to describe the field inside the rotating cylinder is

$$F(r) = J_n(\gamma_n r) , \quad n = 0, \pm 1, \pm 2 \ldots \quad . \tag{38}$$

The complete expression for the transmitted field can be written in the form

$$E_t = E_o \sum_{-\infty}^{\infty} \beta_n (i)^n J_n(\gamma_n r)\, e^{in\phi} . \tag{39}$$

By matching the z-component of the electric fields, as defined by Eqs. (29), (30), and (39), and the ϕ-component of the magnetic field at the boundary

$r = a$ one obtains the following two simultaneous equations:

$$(40) \qquad J_n(k_o a) + \alpha_n H_n^{(1)}(k_o a) = \beta_n J_n(\gamma_n a)$$

and

$$(41) \qquad k_o[J_n'(k_o a) + \alpha_n H_n^{(1)\prime}(k_o a)] = \beta_n \gamma_n J_n'(\gamma_n a) ,$$

where the prime on J and $H^{(1)}$ denotes derivative with respect to the entire argument of these functions.

The solutions for α_n and β_n are

$$(42) \qquad \alpha_n = - \frac{J_n(\rho_o) J_n'(\rho_n) - \frac{k_o}{\gamma_n} J_n(\rho_n) J_n'(\rho_o)}{H_n^{(1)}(\rho_o) J_n'(\rho_n) - \frac{k_o}{\gamma_n} J_n(\rho_n) H_n^{(1)\prime}(\rho_o)}$$

and

$$(43) \qquad \beta_n = - \frac{\frac{k_o}{\gamma_n} [J_n(\rho_o) H_n^{(1)\prime}(\rho_o) - J_n'(\rho_o) H_n^{(1)}(\rho_o)]}{H_n^{(1)}(\rho_o) J_n'(\rho_n) - \frac{k_o}{\gamma_n} J_n(\rho_n) H_n^{(1)\prime}(\rho_o)} ,$$

where

$$\rho_o = k_o a , \quad \rho_n = \gamma_n a .$$

To show the basic difference between the scattering field due to a rotating dielectric cylinder and that due to stationary dielectric cylinder, we will write Eq. (30) in the following form:

$$(44) \qquad E_s = E_o \left[\alpha_o H_o^{(1)}(k_o r) + \sum_{n=1}^{\infty} \alpha_n (i)^n H_n^{(1)}(k_o r) e^{in\phi} + \sum_{n=-1}^{-\infty} \alpha_n (i)^n H_n^{(1)}(k_o r) e^{in\phi} \right]$$

$$= E_o \left[\alpha_o H_o^{(1)}(k_o r) \right.$$

$$\left. + \sum_{n=1}^{\infty} (i)^n H_n^{(1)}(k_o r)(\alpha_n e^{in\phi} + \alpha_{-n} e^{-in\phi}) \right]$$

$$= E_o \left\{ \alpha_o H_o^{(1)}(k_o r) + \sum_{n=1}^{\infty} (i)^n H_n^{(1)}(k_o r) \, . \right.$$

$$\left. . \left[(\alpha_n + \alpha_{-n}) \cos n\phi + i\,(\alpha_n - \alpha_{-n}) \sin n\phi \right] \right\}.$$

Now α_n is numerically different from α_{-n}, hence the scattered field has an asymmetrical part with respect to the direction of incidence, $\phi = 0$. When $\omega_c = 0$ and α_n is equal to α_{-n}, Eq. (44) reduces to the well-known expression for the scattered field from a stationary dielectric cylinder. The nature of the scattered field from a rotating dielectric cylinder is, therefore, similar to the one due to a ferrite rod[5] where the scattered field is also asymmetrical with respect to the direction of incidence.

As far as the results are concerned, the action of the moving medium introduces only a small modification to the scattered field as compared with that from a stationary medium. The formulation described in this report does, however, help us to better understand the mechanism involved in this class of problems.

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